

# Localization of Matters on a String-like Defect

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## Abstract

After presenting string-like solutions with a warp factor to Einstein's equations, we study localization of various spin fields on a string-like defect in a general space-time dimension from the viewpoint of field theory. It is shown that spin 0 and 2 fields are localized on a defect with the exponentially decreasing warp factor. Spin 1 field can be also localized on a defect with the exponentially decreasing warp factor. On the other hand, spin one-half and three-half fields can be localized on a defect with the exponentially increasing warp factor, provided that additional interactions are not introduced. Thus, some mechanism of localization must be invoked for these fermionic fields. These results are very similar to those of a domain wall in five space-time dimensions except the case of spin 1 field.

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# 1 Introduction

It is nowadays believed that the idea of extra dimensions would be one of the most intriguing ideas concerning unification of gauge fields with general relativity. For instance, in superstring theory, the traditional method of the compactification involves writing a ten dimensional manifold as a direct product of a non-compact four dimensional Minkowski space-time and a compact six dimensional Ricci flat manifold (or non-manifold like an orbifold) with the size of the compact space being set by a string scale. Here it is usually assumed that the size of extra spatial dimensions is so small that it is not observed as yet. An important point in this method is that one needs some unknown mechanism of stabilizing the size of all six extra dimensions via non-perturbative string effects. However, it is well known that stabilizing the moduli associated with the size of extra spatial dimensions is a very difficult and unsolved problem [1, 2], which would be a great shortcoming within the present framework of superstring theory.

In recent years, an alternative scenario of the compactification has been put forward [3]. This new idea is based on the possibility that our world is a three brane embedded in a higher dimensional space-time with non-factorizable warped geometry. In this scenario, it is *a priori* assumed that all the matter fields are constrained to live on the three brane, whereas gravity is free to propagate in the extra dimensions. However, this assumption about trapping of Standard Model particles on the brane is not so obvious at first glance. In string theory, such particles could be naturally localized on D3-brane [4], but it is fair to say that string theory realization of the alternative compactification scenario is not completely understood yet. Then it is quite of interest to ask whether field theoretic localization mechanism works as well or not. Indeed, it has been already known that not only scalars and fermions [5] but also gauge fields [6] are localized on the brane in terms of field theoretic mechanism.

This localization mechanism has been recently investigated in  $AdS_5$  space [7, 8, 9, 10, 11]. In particular, in the Randall-Sundrum model in five dimensions [3], the following facts are clarified: spin 0 field is localized on a brane with positive tension which also localizes the graviton [11]. Spin 1 field is not localized neither on a brane with positive tension nor on a brane with negative tension [7, 8]. Moreover, spin 1/2 and 3/2 fields are localized not on a brane with positive tension but on a brane with negative tension [9, 10, 11]. Thus, in order to fulfill the localization of Standard Model particles on a brane with positive tension, it seems that some additional interactions except gravity must be also introduced in the bulk.

The aim of the present article is to extend these works to the case of a string-like defect with codimension 2 in a general space-time dimension. In this respect, it is valuable to ask why we consider the object with codimension 2. One reason is, of course, because it is in itself of interest to generalize many interesting properties obtained in the Randall-Sundrum model in five dimensions to the string-like solutions in general  $D$  dimensions, with  $D = 6$  being of special importance to physics. But the most important reason comes from the observation that logarithmic gauge coupling unification may be achieved in theories with (sets of) two large spatial dimensions [12]. The logarithmic behavior of the Green's functions in effectively two dimensions has a chance of giving rise to logarithmic variation of the parameters on our brane, thereby reproducing the logarithmic running of coupling constants.

The solutions to Einstein's equations in two extra dimensions have been so thus studied by many groups [13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. In this article, we shall present slightly more general solutions with a warp factor in a general space-time dimension.

## 2 Einstein's equations

The action which we consider in this article is that of gravity in general  $D$  dimensions, with the conventional Einstein-Hilbert action and some matter action:

$$S = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-g} (R - 2\Lambda) + \int d^D x \sqrt{-g} L_m, \quad (1)$$

where  $\kappa_D$  denotes the  $D$ -dimensional gravitational constant with a relation  $\kappa_D^2 = 8\pi G_N = \frac{8\pi}{M_*^{D-2}}$  with  $G_N$  and  $M_*$  being the  $D$ -dimensional Newton constant and the  $D$ -dimensional Planck mass scale, respectively. Throughout this article we follow the standard conventions and notations of the textbook of Misner, Thorne and Wheeler [23].

Variation of the action (1) with respect to the  $D$ -dimensional metric tensor  $g_{MN}$  leads to Einstein's equations:

$$R_{MN} - \frac{1}{2}g_{MN}R = -\Lambda g_{MN} + \kappa_D^2 T_{MN}, \quad (2)$$

where the energy-momentum tensor is defined as

$$T_{MN} = -\frac{2}{\sqrt{-g}} \frac{\delta}{\delta g^{MN}} \int d^D x \sqrt{-g} L_m. \quad (3)$$

We shall adopt the following metric ansatz:

$$\begin{aligned} ds^2 &= g_{MN} dx^M dx^N \\ &= g_{\mu\nu} dx^\mu dx^\nu + \tilde{g}_{ab} dx^a dx^b \\ &= e^{-A(r)} \hat{g}_{\mu\nu} dx^\mu dx^\nu + dr^2 + e^{-B(r)} d\Omega_{n-1}^2, \end{aligned} \quad (4)$$

where  $M, N, \dots$  denote  $D$ -dimensional space-time indices,  $\mu, \nu, \dots$  do  $p$ -dimensional brane ones, and  $a, b, \dots$  do  $n$ -dimensional extra spatial ones, so the equality  $D = p + n$  holds. (We assume  $p \geq 4$ .) And  $d\Omega_{n-1}^2$  stands for the metric on a unit  $(n-1)$ -sphere, which is concretely expressed in terms of the angular variables  $\theta_i$  as

$$d\Omega_{n-1}^2 = d\theta_1^2 + \sin^2 \theta_1 d\theta_2^2 + \sin^2 \theta_1 \sin^2 \theta_2 d\theta_3^2 + \dots + \prod_{i=2}^{n-1} \sin^2 \theta_i d\theta_n^2, \quad (5)$$

with the volume element  $\int d\Omega_{n-1} = \frac{2\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2})}$ .

Moreover, we shall take the ansatz for the energy-momentum tensor respecting the spherical symmetry:

$$\begin{aligned} T_\nu^\mu &= \delta_\nu^\mu t_o(r), \\ T_r^r &= t_r(r), \\ T_{\theta_2}^{\theta_2} &= T_{\theta_3}^{\theta_3} = \dots = T_{\theta_n}^{\theta_n} = t_\theta(r), \end{aligned} \quad (6)$$

where  $t_i (i = o, r, \theta)$  are functions of only the radial coordinate  $r$ .

Under these ansatzs, after a straightforward calculation, Einstein's equations reduce to

$$\begin{aligned} e^A \hat{R} - \frac{p(n-1)}{2} A' B' - \frac{p(p-1)}{4} (A')^2 - \frac{(n-1)(n-2)}{4} (B')^2 \\ + (n-1)(n-2) e^B - 2\Lambda + 2\kappa_D^2 t_r = 0, \end{aligned} \quad (7)$$

$$\begin{aligned} e^A \hat{R} + (n-2) B'' - \frac{p(n-2)}{2} A' B' - \frac{(n-1)(n-2)}{4} (B')^2 \\ + (n-2)(n-3) e^B + p A'' - \frac{p(p+1)}{4} (A')^2 - 2\Lambda + 2\kappa_D^2 t_\theta = 0, \end{aligned} \quad (8)$$

$$\begin{aligned} \frac{p-2}{p} e^A \hat{R} + (p-1) (A'' - \frac{n-1}{2} A' B') - \frac{p(p-1)}{4} (A')^2 \\ + (n-1) [B'' - \frac{n}{4} (B')^2 + (n-2) e^B] - 2\Lambda + 2\kappa_D^2 t_o = 0, \end{aligned} \quad (9)$$

where the prime denotes the differentiation with respect to  $r$ , and  $\hat{R}$  is the scalar curvature associated with the brane metric  $\hat{g}_{\mu\nu}$ . Here we define the cosmological constant on the  $(p-1)$ -brane,  $\Lambda_p$ , by the equation

$$\hat{R}_{\mu\nu} - \frac{1}{2} \hat{g}_{\mu\nu} \hat{R} = -\Lambda_p \hat{g}_{\mu\nu}. \quad (10)$$

In addition, the conservation law for the energy-momentum tensor,  $\nabla^M T_{MN} = 0$  takes the form

$$t_r' = \frac{p}{2} A' (t_r - t_o) + \frac{n-1}{2} B' (t_r - t_\theta). \quad (11)$$

It is now known that there are many interesting solutions to these equations (see, for instance, [18]). In this article, we shall confine ourselves to the situation where the geometry has a warp factor, that is,

$$A(r) = cr, \quad (12)$$

where  $c$  is a constant. Before solving a set of the equations, it is useful to notice that for  $n = 1, 2$  Einstein's equations (7), (8), (9) do not include  $e^B$  at all. This fact makes the cases of  $n = 1, 2$  to be quite different from the other higher dimensional cases  $n \geq 3$ . Thus, in what follows, we shall solve a set of the equations in the case of  $n = 2$ .

### 3 String-like solutions (n=2)

Now we shall solve the equations in the case of  $n = 2$ . In this case, under the ansatz (12), Einstein equations (7), (8), (9) are in the form

$$e^{cr} \hat{R} - \frac{p}{2} c B' - \frac{p(p-1)}{4} c^2 - 2\Lambda + 2\kappa_D^2 t_r = 0, \quad (13)$$

$$e^{cr} \hat{R} - \frac{p(p+1)}{4} c^2 - 2\Lambda + 2\kappa_D^2 t_\theta = 0, \quad (14)$$

$$\frac{p-2}{p} e^{cr} \hat{R} - \frac{p-1}{2} c B' - \frac{p(p-1)}{4} c^2 + B'' - \frac{1}{2} (B')^2 - 2\Lambda + 2\kappa_D^2 t_o = 0, \quad (15)$$

and the conservation law takes the form

$$t'_r = \frac{p}{2} c (t_r - t_o) + \frac{1}{2} B' (t_r - t_\theta). \quad (16)$$

From these equations, general solutions can be found as follows:

$$ds^2 = e^{-cr} \hat{g}_{\mu\nu} dx^\mu dx^\nu + dr^2 + e^{-B(r)} d\theta^2, \quad (17)$$

where

$$B(r) = cr + \frac{4}{pc} \kappa_D^2 \int^r dr (t_r - t_\theta), \quad (18)$$

$$\begin{aligned} c^2 &= \frac{1}{p(p+1)} (-8\Lambda + 8\kappa_D^2 \alpha), \\ \hat{R} &= \frac{2p}{p-2} \Lambda_p = -2\kappa_D^2 \beta. \end{aligned} \quad (19)$$

Here  $t_\theta$  must take a definite form, which is given by

$$t_\theta = \beta e^{cr} + \alpha, \quad (20)$$

with  $\alpha$  and  $\beta$  being some constants. Moreover, in order to guarantee the positivity of  $c^2$ ,  $\alpha$  should satisfy an inequality  $-8\Lambda + 8\kappa_D^2 \alpha > 0$ .

It is useful to consider two special cases of the above general solutions. One specific solution is the one without sources ( $t_i = 0$ ). Then we get a special solution found first by Gregory [15, 19]:

$$ds^2 = e^{-cr} \hat{g}_{\mu\nu} dx^\mu dx^\nu + dr^2 + R_0^2 e^{-cr} d\theta^2, \quad (21)$$

with  $R_0$  being a constant. Here the constant  $c$ , the brane scalar curvature and the brane cosmological constant are given by

$$\begin{aligned} c^2 &= \frac{-8\Lambda}{p(p+1)}, \\ \hat{R} &= \frac{2p}{p-2}\Lambda_p = 0. \end{aligned} \quad (22)$$

In this case, as in the corresponding domain wall solution, the bulk geometry is the anti-de Sitter space, and the brane geometry is Ricci-flat with vanishing cosmological constant.

Another specific solution occurs when we have the spontaneous symmetry breakdown  $t_r = -t_\theta$  [18]:

$$ds^2 = e^{-cr} \hat{g}_{\mu\nu} dx^\mu dx^\nu + dr^2 + R_0^2 e^{-c_1 r} d\theta^2, \quad (23)$$

where

$$\begin{aligned} c^2 &= \frac{1}{p(p+1)}(-8\Lambda + 8\kappa_D^2 t_\theta) > 0, \\ c_1 &= c - \frac{8}{pc} \kappa_D^2 t_\theta, \\ \hat{R} &= \frac{2p}{p-2}\Lambda_p = 0. \end{aligned} \quad (24)$$

Notice that this solution is more general than the previous one (21) since this solution reduces to (21) when  $t_\theta = 0$ . This special solution would be utilized to analyse localization of various matters on a string-like defect in the next section.

Finally, let us comment the solutions in general  $n$ . It is straightforward to apply the above calculation procedure to this general case, but as mentioned before the existence of the nontrivial terms involving  $e^B$  prevent interesting solutions like Eq.s (21), (23) from satisfying Einstein's equations. We have checked that for  $n \geq 3$  the solution with the warp factor (12) must be of the form

$$ds^2 = e^{-cr} \hat{g}_{\mu\nu} dx^\mu dx^\nu + dr^2 + R_0^2 d\Omega_{n-1}^2, \quad (25)$$

where

$$\begin{aligned} c^2 &= \frac{-8\Lambda}{p(p+n-2)}, \\ \hat{R} &= \frac{2p}{p-2}\Lambda_p = 0, \end{aligned} \quad (26)$$

where the sources satisfy the relations,  $t_r + (n-1)t_\theta - (n-2)t_o = 0$  and  $t_r = t_o = \text{constant}$ , which are nothing but the relations satisfied in the spontaneous symmetry breakdown [18].

## 4 Localization of various matters

In this section, for clarity we shall limit our attention to a specific string-like solution (23) since the generalization to the general solutions (17) is straightforward. In this paper, we have the physical setup in mind such that 'local cosmic string' sits at the origin  $r = 0$  and then ask the question of whether various bulk fields with spin ranging from 0 to 2 can be localized on the brane by means of only the gravitational interaction. To describe 'local cosmic string' at the origin  $r = 0$ , it is necessary to introduce the boundary conditions meaning that the extra dimensions are conical around the brane with a deficit angle  $\delta$ , which are given by

$$(e^{-\frac{1}{2}B(r)})'|_0^\epsilon = -\frac{\delta}{2\pi}, (e^{-\frac{1}{2}B(0)})' = 1, e^{-B(\epsilon)} = 0, \quad (27)$$

where the boundary conditions are imposed at small radius  $\epsilon$  containing the brane. But these boundary conditions are not directly relevant to the present analysis for the localization. The only relevant information is that the integral over the coordinate  $r$  runs from  $r = 0$  to  $r = \infty$ , whose validity is guaranteed by the above boundary conditions.

### 4.1 Spin 0 scalar field

In this subsection we study localization of a real scalar field in the background geometry (23). It will be shown that provided that the constants  $c$  and  $t_\theta$  simultaneously satisfy certain inequalities, there is a localized zero mode on the string-like defect.

Let us consider the action of a massless real scalar coupled to gravity:

$$S_m = -\frac{1}{2} \int d^D x \sqrt{-g} g^{MN} \partial_M \Phi \partial_N \Phi, \quad (28)$$

from which the equation of motion can be derived:

$$\frac{1}{\sqrt{-g}} \partial_M (\sqrt{-g} g^{MN} \partial_N \Phi) = 0. \quad (29)$$

From now on we shall take  $\hat{g}_{\mu\nu} = \eta_{\mu\nu}$  and define  $P(r) = e^{-cr}$  and  $Q(r) = R_0^2 e^{-c_1 r}$ . In the background metric (23), the equation of motion (29) becomes

$$P^{-1} \eta^{\mu\nu} \partial_\mu \partial_\nu \Phi + P^{-\frac{p}{2}} Q^{-\frac{1}{2}} \partial_r (P^{\frac{p}{2}} Q^{\frac{1}{2}} \partial_r \Phi) + Q^{-1} \partial_\theta^2 \Phi = 0. \quad (30)$$

Let us look for solutions of the form

$$\begin{aligned} \Phi(x^M) &= \phi(x^\mu) \chi(r) \Theta(\theta) \\ &= \phi(x^\mu) \sum_{l,m} \chi_m(r) e^{il\theta}, \end{aligned} \quad (31)$$

where the  $p$ -dimensional scalar field satisfies Klein-Gordon equation

$$\eta^{\mu\nu} \partial_\mu \partial_\nu \phi(x) = m_0^2 \phi(x). \quad (32)$$

Then Eq.(30) reduces to

$$\partial_r^2 \chi_m + \left(\frac{p}{2} \frac{P'}{P} + \frac{1}{2} \frac{Q'}{Q}\right) \partial_r \chi_m + \left(\frac{1}{P} m_0^2 - \frac{1}{Q} l^2\right) \chi_m = 0. \quad (33)$$

It is clear that this equation has the zero-mass ( $m_0 = 0$ ) and  $s$ -wave ( $l = 0$ ) constant solution  $\chi_m(r) = \chi_0 = \text{constant}$ .

Now we wish to show that this constant mode is localized on the defect sitting around the origin  $r = 0$ . The condition for having localized  $p$ -dimensional scalar field is that  $\chi_m(r) = \chi_0$  is normalizable. It is of importance to notice that normalizability of the ground state wave function is equivalent to the condition that the "coupling" constant is nonvanishing. This key observation is fully utilized when we discuss localization of the constant mode of various spin fields in the below.

Let us substitute the zero mode  $\chi_m(r) = \chi_0$  into the starting action (28) and check if the constant solution is a normalizable solution or not. With  $\Phi_0(x^M) = \phi(x^\mu) \chi_0$ , the action (28) can be cast to

$$\begin{aligned} S_m^{(0)} &= -\frac{1}{2} \int d^D x \sqrt{-g} g^{MN} \partial_M \Phi_0 \partial_N \Phi_0 \\ &= -\pi \chi_0^2 \int_0^\infty dr P^{\frac{p}{2}-1} Q^{\frac{1}{2}} \int d^p x \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi. \end{aligned} \quad (34)$$

From this equation, if we define

$$\begin{aligned} I_0 &= \int_0^\infty dr P^{\frac{p}{2}-1} Q^{\frac{1}{2}} \\ &= R_0 \int_0^\infty dr e^{-[(\frac{p}{2}-1)c + \frac{1}{2}c_1]r}, \end{aligned} \quad (35)$$

the condition of having localized  $p$ -dimensional scalar field on the defect requires that  $I_0$  should be finite. Then it is easy to rewrite this condition as inequalities for  $c$  and  $t_\theta$ , whose results are given by

$$\frac{1}{\kappa_D^2} \Lambda < t_\theta < -\frac{p-1}{2\kappa_D^2} \Lambda, \quad (36)$$

for  $c > 0$  and

$$t_\theta > -\frac{p-1}{2\kappa_D^2} \Lambda, \quad (37)$$

for  $c < 0$ . Here for simplicity we have taken the bulk cosmological constant  $\Lambda$  to be negative, in other words, the bulk geometry is anti-de Sitter space. (This assumption is made in this section.) Note that the condition (36) includes the case without sources, (21). This condition also implies that the  $p$ -dimensional scalar field  $\phi$  is localized on the defect with the decreasing warp factor, which exactly corresponds to localization of spin 0 field on a positive-tension brane in the five-dimensional Randall-Sundrum model [11]. We will see in subsection 4.4 that with the same condition as above, spin 2 graviton field is localized on the defect.



## 4.2 Spin 1 vector field

It was shown in the Randall-Sundrum model in  $AdS_5$  space that spin 1 vector field is not localized neither on a brane with positive tension nor on a brane with negative tension so the Dvali-Shifman mechanism [6] must be invoked for the vector field localization [11]. Remarkably, it will be shown in this subsection that spin 1 vector field *is* localized on a string-like defect, which is in sharp contrast with the domain wall case. So we do not need to introduce additional mechanism for the vector field localization in the case at hand.

Let us start with the action of  $U(1)$  vector field:

$$S_m = -\frac{1}{4} \int d^D x \sqrt{-g} g^{MN} g^{RS} F_{MR} F_{NS}, \quad (38)$$

where  $F_{MN} = \partial_M A_N - \partial_N A_M$  as usual. From this action the equation motion is given by

$$\frac{1}{\sqrt{-g}} \partial_M (\sqrt{-g} g^{MN} g^{RS} F_{NS}) = 0. \quad (39)$$

In the background metric (23), this equation is reduced to

$$P^{-1} \eta^{\mu\nu} g^{MN} \partial_\mu F_{\nu N} + P^{-\frac{p}{2}} Q^{-\frac{1}{2}} \partial_r (P^{\frac{p}{2}} Q^{\frac{1}{2}} g^{MN} F_{rN}) + Q^{-1} g^{MN} \partial_\theta F_{\theta N} = 0. \quad (40)$$

By choosing the gauge condition  $A_\theta = 0$  and decomposing the vector field as

$$\begin{aligned} A_\mu(x^M) &= a_\mu(x^\mu) \sum_{l,m} \rho_m(r) e^{il\theta}, \\ A_r(x^M) &= a_r(x^\mu) \sum_{l,m} \rho_m(r) e^{il\theta}, \end{aligned} \quad (41)$$

it is straightforward to see that there is the  $s$ -wave ( $l = 0$ ) constant solution  $\rho_m(r) = \rho_0 = \text{constant}$  and  $a_r = \text{constant}$ . Note that in deriving this solution we have used  $\partial_\mu a^\mu = \partial^\mu f_{\mu\nu} = 0$  with the definition of  $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ .

As in the scalar field in the previous subsection, let us substitute this constant solution into the action (38) in order to see if the solution is a normalizable solution or not. It turns out that the action is reduced to

$$\begin{aligned} S_m^{(0)} &= -\frac{1}{4} \int d^D x \sqrt{-g} g^{MN} g^{RS} F_{MR}^{(0)} F_{NS}^{(0)} \\ &= -\frac{\pi}{2} \rho_0^2 \int_0^\infty dr P^{\frac{p}{2}-2} Q^{\frac{1}{2}} \int d^p x \eta^{\mu\nu} \eta^{\lambda\sigma} f_{\mu\lambda} f_{\nu\sigma}. \end{aligned} \quad (42)$$

Provided that we define  $I_1$  by the equation

$$\begin{aligned} I_1 &= \int_0^\infty dr P^{\frac{p}{2}-2} Q^{\frac{1}{2}} \\ &= R_0 \int_0^\infty dr e^{-[(\frac{p}{2}-2)c + \frac{1}{2}c_1]r}, \end{aligned} \quad (43)$$

the condition of having localized  $p$ -dimensional vector field on the defect requires  $I_1$  to be finite. This condition can be expressed as

$$\frac{1}{\kappa_D^2}\Lambda < t_\theta < -\frac{p-3}{4\kappa_D^2}\Lambda, \quad (44)$$

for  $c > 0$  and

$$t_\theta > -\frac{p-3}{4\kappa_D^2}\Lambda, \quad (45)$$

for  $c < 0$ . Thus, the vector field can be localized on the string-like defect with the exponentially decreasing warp factor  $c > 0$  by selecting the source  $t_\theta$  such that the inequality (44) should be satisfied. This result is quite different from that of the vector field in the domain wall in  $AdS_5$  [7, 8, 11] where there was no localized solutions. The difference with the domain wall case is that the background metric (23) now contains the factor  $e^{-c_1 r}$  in the angular part, which gives us this nontrivial and interesting result. Indeed, if we set  $p = 4$  in  $I_1$ , the contribution from  $c$  vanishes and  $I_1$  becomes dependent on only  $c_1$ . However, this term is trivially zero in the domain wall case, thereby giving rise to a divergent quantity irrespective of  $c$  in the case of the domain wall solution. Accordingly, we do not need to invoke some additional mechanism for localization of the vector field in the case at hand.

### 4.3 Spin 1/2 fermionic field

Now we are ready to consider spin 1/2 fermion. Our starting action is the Dirac action given by

$$S_m = \int d^D x \sqrt{-g} \bar{\Psi} i \Gamma^M D_M \Psi, \quad (46)$$

from which the equation of motion is given by

$$0 = \Gamma^M D_M \Psi = (\Gamma^\mu D_\mu + \Gamma^r D_r + \Gamma^\theta D_\theta) \Psi. \quad (47)$$

We shall introduce the vielbein  $e_M^{\bar{M}}$  (and its inverse  $e_M^{\bar{M}}$ ) through the usual definition  $g_{MN} = e_M^{\bar{M}} e_N^{\bar{N}} \eta_{\bar{M}\bar{N}}$  where  $\bar{M}, \bar{N}, \dots$  denote the local Lorentz indices. From the formula  $\Gamma^M = e_M^{\bar{M}} \gamma^{\bar{M}}$  with  $\Gamma^M$  and  $\gamma^{\bar{M}}$  being the curved gamma matrices and the flat gamma ones, respectively, we have the relations:

$$\Gamma^\mu = P^{-\frac{1}{2}} \gamma^\mu, \quad \Gamma^r = \gamma^r, \quad \Gamma^\theta = Q^{-\frac{1}{2}} \gamma^\theta. \quad (48)$$

The spin connection  $\omega_M^{\bar{M}\bar{N}}$  in the covariant derivative  $D_M \Psi = (\partial_M + \frac{1}{4} \omega_M^{\bar{M}\bar{N}} \gamma_{\bar{M}\bar{N}}) \Psi$  is defined as

$$\begin{aligned} \omega_M^{\bar{M}\bar{N}} = & \frac{1}{2} e^{N\bar{M}} (\partial_M e_N^{\bar{N}} - \partial_N e_M^{\bar{N}}) - \frac{1}{2} e^{N\bar{N}} (\partial_M e_N^{\bar{M}} - \partial_N e_M^{\bar{M}}) \\ & - \frac{1}{2} e^{P\bar{M}} e^{Q\bar{N}} (\partial_P e_{Q\bar{R}} - \partial_Q e_{P\bar{R}}) e_M^{\bar{R}}, \end{aligned} \quad (49)$$

so the nonvanishing components are evaluated for the background metric (23):

$$\omega_{\theta}^{\bar{r}\bar{\theta}} = -\frac{1}{2}Q^{-\frac{1}{2}}Q', \quad \omega_{\mu}^{\bar{r}\bar{\mu}} = -\frac{1}{2}P^{-\frac{1}{2}}P'\delta_{\mu}^{\bar{\mu}}. \quad (50)$$

Therefore, the covariant derivative can be calculated to

$$D_{\mu}\Psi = (\partial_{\mu} - \frac{1}{4}\frac{P'}{P}\Gamma_r\Gamma_{\mu})\Psi, \quad D_r\Psi = \partial_r\Psi, \quad D_{\theta}\Psi = (\partial_{\theta} - \frac{1}{4}\frac{Q'}{Q}\Gamma_r\Gamma_{\theta})\Psi. \quad (51)$$

Substituting Eq.(51) in the equation of motion (47), we will search for the solutions of the form  $\Psi(x^M) = \psi(x^{\mu})\alpha(r)\Sigma e^{il\theta}$  where  $\psi(x^{\mu})$  satisfies the massless  $p$ -dimensional Dirac equation  $\gamma^{\mu}\partial_{\mu}\psi = 0$ . Then the equation of motion (47) is reduced to

$$(\partial_r + \frac{p}{4}\frac{P'}{P} + \frac{1}{4}\frac{Q'}{Q})\alpha(r) = 0. \quad (52)$$

The solution to this equation reads:

$$\alpha(r) = c_2 P^{-\frac{p}{4}} Q^{-\frac{1}{4}}, \quad (53)$$

with  $c_2$  being an integration constant. Here we have considered the  $s$ -wave solution.

Now let us show that the solution (53) is normalizable if we use the exponentially not decreasing but increasing warp factor.

$$\begin{aligned} S_m^{(0)} &= \int d^D x \sqrt{-g} \bar{\Psi}_0 i \Gamma^M D_M \Psi_0 \\ &= 2\pi \int_0^{\infty} dr P^{\frac{p}{2}-\frac{1}{2}} Q^{\frac{1}{2}} \alpha(r)^2 \int d^p x \bar{\psi} i \gamma^{\mu} \partial_{\mu} \psi + \dots \end{aligned} \quad (54)$$

In order to localize spin 1/2 fermion in this framework, the integral  $I_{\frac{1}{2}}$ , which is defined as

$$\begin{aligned} I_{\frac{1}{2}} &= \int_0^{\infty} dr P^{\frac{p}{2}-\frac{1}{2}} Q^{\frac{1}{2}} \alpha(r)^2 \\ &= c_2^2 \int_0^{\infty} dr e^{\frac{1}{2}cr}, \end{aligned} \quad (55)$$

should be finite. But this quantity is obviously divergent for  $c > 0$  while it is finite for  $c < 0$ . This situation is the same as in the domain wall in the Randall-Sundrum framework [11] where for localization of spin 1/2 field additional localization method by Jackiw and Rebbi [24] was introduced. This method could be applied even to the present situation, but we believe that the most natural and interesting approach would be to construct a supergravity theory corresponding to the situation at hand.

#### 4.4 Spin 3/2 fermionic field

Next we turn to spin 3/2 field, in other words, the gravitino. We will encounter the same result as in spin 1/2 field.

Let us begin with the action of the Rarita-Schwinger gravitino field:

$$S_m = \int d^D x \sqrt{-g} \bar{\Psi}_M i \Gamma^{[M} \Gamma^N \Gamma^{R]} D_N \Psi_R, \quad (56)$$

from which the equation of motion is given by

$$\Gamma^{[M} \Gamma^N \Gamma^{R]} D_N \Psi_R = 0. \quad (57)$$

Here the square bracket denotes the anti-symmetrization and the covariant derivative is defined with the affine connection  $\Gamma_{MN}^R = e_M^R (\partial_M e_N^{\bar{M}} + \omega_M^{\bar{M}\bar{N}} e_{N\bar{N}})$  by

$$D_M \Psi_N = \partial_M \Psi_N - \Gamma_{MN}^R \Psi_R + \frac{1}{4} \omega_M^{\bar{M}\bar{N}} \gamma_{\bar{M}\bar{N}} \Psi_N. \quad (58)$$

After taking the gauge condition  $\Psi_\theta = 0$ , the nontrivial components of the covariant derivative are easily calculated:

$$\begin{aligned} D_\mu \Psi_\nu &= \partial_\mu \Psi_\nu + \frac{1}{2} P' \eta_{\mu\nu} \Psi_r - \frac{1}{4} \frac{P'}{P} \Gamma_r \Gamma_\mu \Psi_\nu, \\ D_\mu \Psi_r &= \partial_\mu \Psi_r - \frac{1}{2} \frac{P'}{P} \Psi_\mu - \frac{1}{4} \frac{P'}{P} \Gamma_r \Gamma_\mu \Psi_r, \\ D_r \Psi_\mu &= \partial_r \Psi_\mu - \frac{1}{2} \frac{P'}{P} \Psi_\mu, \\ D_r \Psi_r &= \partial_r \Psi_r, \\ D_\theta \Psi_\mu &= \partial_\theta \Psi_\mu - \frac{1}{4} \frac{Q'}{Q} \Gamma_r \Gamma_\theta \Psi_\mu, \\ D_\theta \Psi_r &= \partial_\theta \Psi_r - \frac{1}{4} \frac{Q'}{Q} \Gamma_r \Gamma_\theta \Psi_r, \\ D_\theta \Psi_\theta &= \frac{1}{2} Q' \Psi_r. \end{aligned} \quad (59)$$

Substituting Eq.(59) in the equation of motion (57), we will look for the solutions of the form  $\Psi_\mu(x^M) = \psi_\mu(x^\mu) u(r) \sum e^{il\theta}$ ,  $\Psi_r(x^M) = \psi_r(x^\mu) u(r) \sum e^{il\theta}$  where  $\psi_\mu(x^\mu)$  satisfies the following  $p$ -dimensional equations  $\gamma^\mu \psi_\mu = \partial^\mu \psi_\mu = \gamma^{[\mu} \gamma^\nu \gamma^{\rho]} \partial_\nu \psi_\rho = 0$ . Then the equation of motion (57) is of the form

$$(\partial_r + \frac{p-2}{4} \frac{P'}{P} + \frac{1}{4} \frac{Q'}{Q}) u(r) = 0. \quad (60)$$

The solution to this equation reads:

$$u(r) = c_3 P^{-\frac{p-2}{4}} Q^{-\frac{1}{4}}, \quad (61)$$

with  $c_3$  being an integration constant. Again in the above we have considered the  $s$ -wave solution and  $\psi_r = 0$ .

We shall show that as in the case of spin 1/2 field the solution (61) is normalizable if we use the exponentially increasing warp factor.

$$\begin{aligned} S_m^{(0)} &= \int d^D x \sqrt{-g} \bar{\Psi}_M^{(0)} i \Gamma^{[M} \Gamma^N \Gamma^{R]} D_N \Psi_R^{(0)} \\ &= 2\pi \int_0^\infty dr P^{\frac{p}{2}-\frac{3}{2}} Q^{\frac{1}{2}} u(r)^2 \int d^p x \bar{\psi}_\mu i \gamma^{[\mu} \gamma^\nu \gamma^{\rho]} \partial_\nu \psi_\rho. \end{aligned} \quad (62)$$

In order to localize spin 3/2 fermion, the integral  $I_{\frac{3}{2}}$ , which is defined as

$$\begin{aligned} I_{\frac{3}{2}} &= \int_0^\infty dr P^{\frac{p}{2}-\frac{3}{2}} Q^{\frac{1}{2}} u(r)^2 \\ &= c_3^2 \int_0^\infty dr e^{\frac{1}{2}cr}, \end{aligned} \quad (63)$$

must be finite. But this expression is equivalent to  $I_{\frac{1}{2}}$  up to an overall constant factor so it is divergent for  $c > 0$  while it is finite for  $c < 0$ . This result is also the same as in the domain wall in the Randall-Sundrum framework [11].

## 4.5 Spin 2 field

For the sake of completeness we briefly touch on spin 2 graviton field from our approach since this case has been already examined in Ref.[19].

Let us consider the following metric fluctuations:

$$\begin{aligned} ds^2 &= g_{MN} dx^M dx^N \\ &= g_{\mu\nu} dx^\mu dx^\nu + \tilde{g}_{ab} dx^a dx^b \\ &= e^{-cr} (\eta_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu + dr^2 + R_0^2 e^{-c_1 r} d\theta^2. \end{aligned} \quad (64)$$

Then the equation of motion for the fluctuations  $h_{\mu\nu}$  is found to be:

$$\frac{1}{\sqrt{-g}} \partial_M (\sqrt{-g} g^{MN} \partial_N h_{\mu\nu}) = 0. \quad (65)$$

Consequently, it turns out that the equation of motion for the fluctuations in the present background becomes equivalent to that of the scalar field considered in subsection 4.1 [25, 17]. Accordingly, we expect that the condition for localization of spin 2 field might be equivalent to that of spin 0 field. This is indeed the case as shown in what follows.

Let us look for solutions of the form

$$\begin{aligned} h_{\mu\nu}(x^M) &= \check{h}_{\mu\nu}(x^\mu) \varphi(r) \Theta(\theta) \\ &= \check{h}_{\mu\nu}(x^\mu) \sum_{l,m} \varphi_m(r) e^{il\theta}, \end{aligned} \quad (66)$$

where  $\eta^{\mu\nu}\partial_\mu\partial_\nu\check{h}_{\rho\sigma} = m_0^2\check{h}_{\rho\sigma}$ . It is then easy to show that the equation of motion has the zero-mass ( $m_0 = 0$ ) and  $s$ -wave ( $l = 0$ ) constant solution  $\varphi_m(r) = \varphi_0 = \text{constant}$ . Substitution of this zero mode  $\varphi_m(r) = \varphi_0$  into the Einstein-Hilbert action (1) leads to

$$S^{(0)} \sim \varphi_0^2 \int_0^\infty dr P^{\frac{p}{2}-1} Q^{\frac{1}{2}} \int d^p x \partial^\rho \check{h}^{\mu\nu} \partial_\rho \check{h}_{\mu\nu} + \dots \quad (67)$$

From this equation, if we define  $I_2$  by

$$I_2 = \int_0^\infty dr P^{\frac{p}{2}-1} Q^{\frac{1}{2}}, \quad (68)$$

the condition of having localized  $p$ -dimensional graviton field on the defect requires that  $I_2$  should be finite. Note that  $I_2 = I_0$ , so that we have the same result for localization of the graviton as in the spin 0 scalar field.

## 5 Discussions

In this paper, we have investigated two problems, those are, finding solutions with the warp factor corresponding to string-like defects and checking localization of various spin fields on such a string-like defect from the viewpoint of field theory. We have presented more general solutions compared to the solutions found so far. Moreover, it has been found that spin 0 and 2 fields are localized on a defect with the exponentially decreasing warp factor, and spin 1 field can be also localized on a defect with the exponentially decreasing warp factor by selecting an appropriate range of values of sources. On the contrary, spin 1/2 and 3/2 fields can be localized on a defect with the exponentially increasing warp factor.

These results for localization of various spin fields coincide with the corresponding ones [11] in the Randall-Sundrum model [3] and many brane model [26, 27] except spin 1 vector field. It is remarkable that there is no localized vector field on the brane in the domain wall model, whereas vector field can be localized on the defect in the string-like model owing to the existence of the nontrivial exponential factor in the angular part of the metric.

Localizing the fermionic degrees of freedom on the brane or the defect requires us to introduce other interactions but gravity. The most natural approach for it seems to embed the present model in the supergravity theory. In this respect, it is worthwhile to emphasize that six-dimensional supergravity is more beautiful than five-dimensional supergravity. Recently, the authors in Ref.[28] have studied a covariant formalism of six-dimensional supergravity. In future, we wish to extend the present model to the supergravity model.

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